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A proposal for the generation of pure quantum number states

A Rostami
E Eng Dept. No 8 Bld , Faculty of Engineering, Tabriz University, Tabriz 51664, Iran
E-mail Rostami@ark tabrizu.ac ir

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Abstract : The superposition of coherent states can be used for the generation of pure quantum number states Also, in this paper shown that, the limited number of coherent states sufficient for this approximation For the approximation of number states according to n ($|n>$), the n plus one component of coherent states must be used which related together in phase or amplitude of the coherent state parameters

Keywords : Superposition of coherent states, pure number state, quantum state generation

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1. Introduction

The role of the Heisenberg uncertainty principle as the quantum mechanical limit on the precision of measurments has been discussed for many years [1–4]. Many example in uncertainty principle range, such as superconducting quantum interferometric device (SQUID) amplifier which is close to quantum limited sensitivity, measurment of optical fields and the transmission of information using a coherent laser radiation in optical fiber, recently needed for applications in research area. For these applications the sensitivity of the measurments must be more than the uncertainty principle limit. Then for this requirements, the quantum limit must be bypassed such as classical phyics. But this limit can not be removed at least from the quantum mechanical point of view. For creation of this possibility (High precision measurments), one must be found the quantum states nearest to the classical situation (Minimum Uncertainty States). These states named squeezed states and discussed more so far [5–11]. But all studies are restricted to quantum scale [12–16] and very few papers can be found in macroscopic approach for generation of these states. In my previous paper [11], the macroscopic approach for the study of squeezed states, has been investigated. In that paper, the superposition of coherent states can be used for the generation of squeezed states. In the quantum optics, the pure number states are important from the theoretical and experimental point of view. Generation and detection of these states have the critical roles in the quantum optical engineering. For this purpose, the generation of pure quantum number states is the subject of the paper. Here, we have shown that by use of the limited number of coherent states, one can approximate the pure number states.

2. Pure quantum number state generation

In this section, investigation of the generation possibility of the pure states in number base ($|n>$) has been made. For this, the superposition of coherent states idea can be examined for the generation of these states. In this approach, the superposition of limited number of coherent states can be studied as following :

Case (a) : In this case, one tries to choose the same coherent state parameters in absolute value but different in phases. The phase of the coherent state parameters must have equal distance as follows :

$$|\psi\rangle = \sum_{i=0}^{N} C_i |\alpha_i\rangle, \tag{1}$$

where $|\alpha_i>$ is given as

$$|\alpha_i\rangle = \| \alpha | e^{\frac{j2\pi i}{N+1}} \rangle, \quad i = 0, \ldots, N.$$

For creating the pure number states, the proposed method must have the unity projection in desired direction $(|N\rangle)$

and about zero value in the other directions. For this purpose, the expansion coefficient (C_i in eq. (1)) must be apropriately chosen which are obtained from the following relations.

$$|\alpha_i\rangle = e^{-\frac{|\alpha_i|^2}{2}} \sum_{m=0}^{\infty} \frac{\alpha_i^m}{\sqrt{m!}} |m\rangle$$

By inserting the above relation for coherent state in eq. (1), one obtains the following relations :

$$|\psi\rangle = \sum_{m=0}^{\infty} \left[\sum_{i=0}^{N} C_i e^{-\frac{|\alpha|^2}{2}} \frac{|\alpha|^m}{\sqrt{m!}} e^{i\frac{2\pi m i}{N+1}} \right] |m\rangle$$

For obtaining the C_i coefficient, one must examine the projection of this wave function in the desired direction as follows :

$$\langle N|\psi\rangle = \sum_{i=0}^{N} C_i \frac{e^{-|\alpha|^2/2} |\alpha|^N}{\sqrt{N!}} e^{i\frac{2\pi N i}{N+1}}. \qquad (2)$$

In this relation, C_i must be chosen such that the $\langle N|\psi\rangle$ approaches to unity. Thus for better approximation, the expansion coefficient (C_i) must be chosen as :

$$C_i = \frac{e^{|\alpha|^2/2} \sqrt{N!}}{(N+1)|\alpha|^N} e^{-i\frac{2\pi N i}{N+1}}.$$

With this value for C_i, the wave function given in eq. (1) is as follows :

$$|\psi\rangle = \sum_{m=0}^{\infty} \left[\sum_{i=0}^{N} \frac{|\alpha|^{m-N}}{n+1} \sqrt{\frac{N!}{m!}} e^{j\frac{2\pi i(m-N)}{N+1}} \right] |m\rangle$$

or $$|\psi\rangle = \sum_{m=0}^{\infty} d_m |m\rangle.$$

The coefficient d_m illustrated in Figure 1. According to this diagram, one can write the wave function as follows :

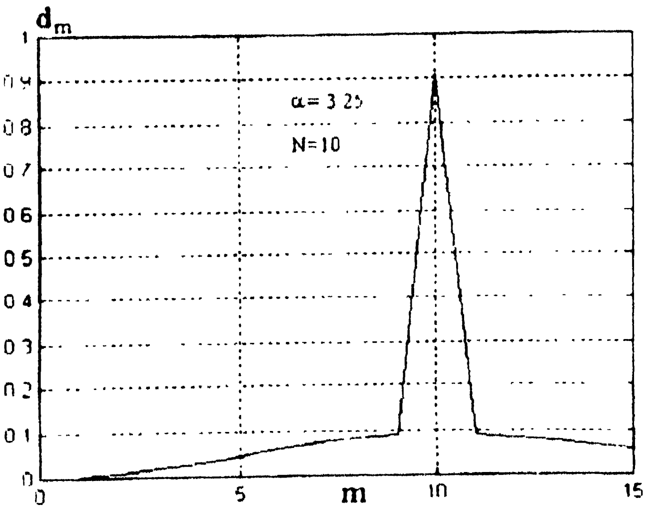

Figure 1. The wave function coefficient *vs* to *m* (First Method)

$$|\psi\rangle = \ldots + d_{N-1}|N-1\rangle + d_N|N\rangle + d_{N+1}|N+1\rangle + \ldots,$$

where $|d_{N-1}|^2$ or $|d_{N+1}|^2$ are the errors for approximation and the results shown in Figure 1 show that the $|\psi\rangle$ has appropriately approximated the $|N\rangle$ in number base.

Case (b) : In this case, one attempts to choose the same coherent state parameters in phases but different in absolute value. The magnitude of coherent state parameters adopted specially according to equal deviation about central parameter (α_0), are

$$|\psi\rangle = \sum_{i=0}^{N} C_i |\alpha_i\rangle \qquad (3)$$

where $|\alpha_i\rangle$ are given as follows

$$|\alpha_i\rangle = |\alpha_0 - \frac{N}{2}\delta + i\delta\rangle, \quad i = 0, \ldots N,$$

where δ is the distance between two adjacent coherent states parameters. For obtaining the pure number state, one must be able to obtain the probability density $|\psi\rangle$ in the mentioned pure state and also must approximately approche to 1 or projection in this direction must be unity and in other direction, must have zero component. For this purpose, the expansion coefficient (C_i in eq. (3)) must be apropriately chosen which obtained in the following relations :

$$|\alpha_i\rangle = e^{-\frac{|\alpha_i|^2}{2}} \sum_{m=0}^{\infty} \frac{\alpha_i^m}{\sqrt{m!}} |m\rangle.$$

By inserting this relation in eq. (3), the state function is

$$|\psi\rangle = \sum_{m=0}^{\infty} \left[\sum_{i=0}^{N} C_i e^{-\frac{\left|\alpha_0 - \frac{N}{2}\delta + i\delta\right|^2}{2}} \frac{\left(\alpha_0 - \frac{N}{2}\delta + i\delta\right)^m}{\sqrt{m!}} \right] |m\rangle$$

For obtaining the C_i coefficients, one must calculate the projection of state function in the desired direction as follows :

$$\langle N|\psi\rangle = \sum_{i=0}^{N} C_i e^{-\frac{\left|\alpha_0 - \frac{N}{2}\delta + i\delta\right|^2}{2}} \frac{\left(\alpha_0 - \frac{N}{2}\delta + i\delta\right)^N}{\sqrt{N!}}. \qquad (4)$$

According to the above component in the desired direction, C_i must adopt such values that the $< N|\psi >$ approaches to unity. Thus we have

$$C_i = e^{\frac{\left|\alpha_0 - \frac{N}{2}\delta + i\delta\right|^2}{2}} \frac{\sqrt{N!}}{(N+1)\left(\alpha_0 - \frac{N}{2}\delta + i\delta\right)^N}.$$

With this value for C_i, the wave function in eq. (3) can written as

$$|\psi\rangle = \sum_{m=0}^{\infty}\left[\sum_{i=0}^{N}\frac{1}{N+1}\sqrt{\frac{N!}{m!}}\frac{\left(\alpha_0 - \frac{N}{2}\delta + i\delta\right)^m}{\left(\alpha_0 - \frac{N}{2}\delta + i\delta\right)^N}\right]|m\rangle$$

or $\quad |\psi\rangle = \sum_{m=0}^{\infty} d_m |m\rangle,$

where d_m is

$$d_m = \sum_{i=0}^{N}\frac{1}{N+1}\sqrt{\frac{N!}{m!}}\frac{\left(\alpha_0 - \frac{N}{2}\delta + i\delta\right)^m}{\left(\alpha_0 - \frac{N}{2}\delta + i\delta\right)^N}.$$

The coefficient d_m given in Figure 2. Also, according to this diagram, the state function can be writen as

$$|\psi\rangle = \ldots + d_{N-1}|N-1\rangle + d_N|N\rangle + d_{N+1}|N+1\rangle + \ldots$$

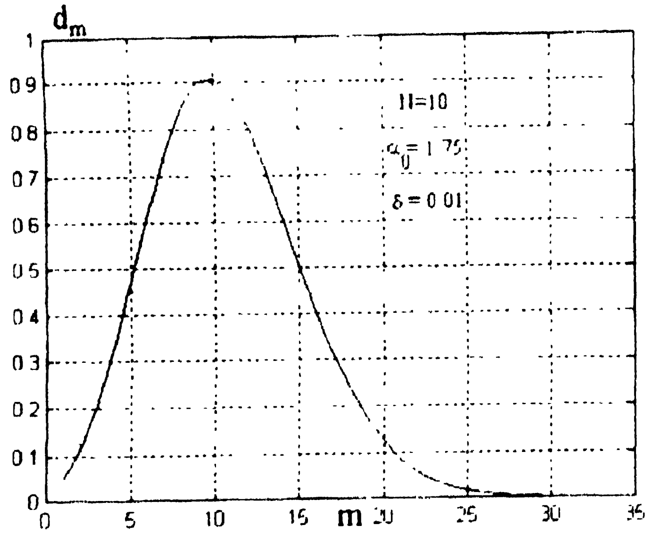

Figure 2. The wave function coefficient *vs* to *m* (Second Method)

which appropriately approximate the state function for $|N\rangle$. Also, d_{N-1} and d_{N+1} is the error for approximation. For implementation of these methods the semiconductor laser diode array for creation of equal distance coherent states can be proposed. Bias current can be controlled the intensity of lasers and easily realize the linear combination of equal distance coherent states.

3. Conclusion

Nonclassical properties of linear combination of coherent states are used for the generation of number base states. Also in this paper, the two approaches for realization of pure quantum number state based on superposition of limited number coherent states have been illustrated and the laser diode array for creation of this state is proposed.

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